

THE CHEMICAL EVOLUTION OF THE SOLAR NEIGHBOURHOOD

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Abstract Recent models of galactic chemical evolution account for updated evolutionary models of massive stars (with special emphasis on stellar winds) and for the effects of intermediate mass and massive binaries. The results are summarised. We also present a critical discussion on possible effects of stellar rotation on overall galactic chemical evolutionary simulations.

1. INTRODUCTION

Most of the chemical evolutionary models (CEMs) consider the enrichment of the interstellar medium (ISM) by single stars only (with some parametrisation to account for the effects of type Ia supernovae). However, there is overwhelming evidence that a large fraction of all stars has a close companion (for reviews, see Mermilliod, 2001; Van Rensbergen, 2001; Mason et al., 2001 and references therein) and a discussion of the effects of binaries on CEMs became necessary. This was the main topic of an extended study in Brussels that was published in De Donder and Vanbeveren (2002). The results are summarised in Sect. 2.

CEMs of galaxies rely on stellar evolution and therefore improvements in the physics of the latter may have important consequences for the former. To illustrate, Sect. 3 considers recent updates of the Wolf-Rayet (WR) type stellar wind mass loss in massive stars and the effect on stellar evolution (remind that WR stars are massive hydrogen deficient core helium burning stars, further abbreviated as CHeB). The consequences for CEM simulations will be discussed in Sect. 4.

Meynet and Maeder (2000) concluded that stellar rotation modifies all the outputs of stellar evolution of massive stars (see also Heger et al.,

2000). In Sect. 5 we present a critical discussion and we estimate the effect of rotation on CEMs.

2. INTERACTING BINARIES AND THE EFFECT ON CHEMICAL EVOLUTION

To study the effects of binaries on CEMs, it is essential to combine a star formation model, a galaxy formation model, a galactic evolutionary model and a population number synthesis (PNS) model which accounts for single stars and interacting binaries. A description of our PNS code can be found in Vanbeveren et al. (1998a, b, c), Vanbeveren (2000, 2001), De Donder and Vanbeveren (2002). In its present state it allows to explore a population of binaries with initial parameters $3 \leq M_1/M_\odot \leq 120$ (M_1 = primary mass), $0 < q \leq 1$ (q = binary mass ratio = mass secondary/mass primary) and $1 \text{ day} \leq P \leq 10 \text{ years}$ (P = binary orbital period), and a population of single stars with $0.1 \leq M/M_\odot \leq 120$. It accounts for the RLOF, mass transfer and mass accretion in case A and case Br binaries, the common envelope (CE) process and spiral-in in case Bc and case C binaries, the CE and spiral-in in binaries with a compact companion (a white dwarf, a neutron star or a black hole) and the effects of an asymmetric SN explosion on the binary parameters when one of the binary components explode. We use a detailed set of stellar evolutionary calculations for different initial metallicities ($0.001 \leq Z \leq 0.02$) where, in the case of massive stars, the most recent stellar wind mass loss rate formalisms are implemented (e.g. Sect. 3 for the WR- \dot{M} formalism). Type Ia supernovae (SN Ia) are associated with the single degenerate scenario (Hachisu et al., 1996) and, separately, with the double degenerate scenario (Iben and Tutukov, 1984; Webbink, 1984) of mass accreting white dwarfs (notice that the only realistic way to study the effects on CEMs of the Fe-enrichment of SN Ia's is to link a detailed PNS model with both scenario's). All our evolutionary calculations rely on the core overshooting formalism proposed by the Geneva group (Schaller et al., 1992).

The results of the CEM simulations with binaries performed in Brussels have been published in De Donder and Vanbeveren (2002). Summarising:

- although binaries return less matter to the ISM than single stars, the temporal evolution of the rate of star formation only marginally depends on whether binaries form or not
- the age-metallicity relation (AMR) shows the time evolution of the ratio $[\text{Fe}/\text{H}]$ which is usually taken as a measure of the metallic-

ity of the galaxy. We concluded that the AMR predicted by a theoretical galactic model hardly depends on whether binaries are included or not

- the observed solar carbon abundance and the observational behaviour of the ratio $[C/Fe]$ as function of $[Fe/H]$ in the solar neighbourhood may be indirect evidence for the presence of a significant intermediate mass binary population during the whole evolution of the solar region
- the temporal evolution of the elements 4He , ${}^{16}O$, ${}^{20}Ne$, ${}^{24}Mg$, ${}^{28}Si$, ${}^{32}S$ and ${}^{40}Ca$ predicted by a CEM where the effect of binaries is considered in detail differ by no more than a factor of two to three from the results of models where all stars are treated as single stars and where the effect of binaries is simulated only to account for the SN Ia population.

The latter obviously means that as long as observational and theoretical uncertainty is at least a factor two, the sophisticated CEMs including binaries are as good (or as bad) as those without binaries.

3. A WR-TYPE \dot{M} -FORMALISM AND THE EFFECT ON MASSIVE STAR EVOLUTION

The formalism. Using a hydrodynamic atmosphere code where the stellar wind is assumed to be homogeneous, Hamann et al. (1995) determined \dot{M} -values for a large number of WR stars. Since then evidence has grown that these winds are clumpy and that a homogeneous model overestimates \dot{M} , typically by a factor 2-4 (see Hamann and Koesterke, 1998, and references therein). In the period 1997-1998 massive single star and binary evolutionary calculations were published (Vanbeveren et al., 1998a, b, c) where we adopted the lower WR mass loss rates as inferred from the studies listed above. We assumed a (simple) relation between \dot{M} (M_{\odot}/yr) and the stellar luminosity L (in L_{\odot})

$$\log(-\dot{M}) = a \log L - b \quad (1.1)$$

and we tried to find appropriate values for the constants a and b by accounting for the following criteria and observations known at that time:

- the WN5 star HD 50896 (WR 6) has a luminosity $\log L = 5.6-5.7$ and a $\log(-\dot{M}) \approx -4.4$ (Schmutz, 1997)
- the $\log(-\dot{M})$ of the WNE component of the binary V444 Cyg (WR 139) derived from the observed orbital period variation is ≈ -5

(Underhill et al., 1990). Its orbital mass is $9 M_{\odot}$ and using a mass-luminosity relation holding for WNE-binary components (Vanbevereren and Packet, 1979; Langer, 1989) it follows that its $\log L = 5$

- the observed masses of black hole (BH) components in X-ray binaries indicate that stars with initial mass $\geq 40 M_{\odot}$ should end their life with a mass larger than $10 M_{\odot}$ (= the mass of the star at the end of CHeB)
- the WN/WC number ratio predicted by stellar evolution depends on the WR-type \dot{M} -formalism. Therefore, last but not least, we looked for a and b values which predict the observed WN/WC number ratio (≈ 1) for the solar neighbourhood.

This exercise allowed us to propose the following relation:

$$\log(-\dot{M}) = \log L - 10 \quad (1.2)$$

Since then more WR stars have been investigated with detailed atmosphere codes including the effects of clumping. They are listed in Van Bever and Vanbevereren (2002) and we concluded that Equation 1.2 still fits fairly well these new observations. Interestingly, the WC6 star OB10-WR1 in the association OB10 of M31 has been investigated by Smart et al. (2001) and also fits Equation 1.2.

Nugis and Lamers (2000, NL) proposed an alternative WR mass loss relation based on a large sample of Galactic WR stars. It can easily be checked that NL is not more accurate than our relation.

The effect of metallicity Z on \dot{M} . The Z -dependency of WR mass loss rates deserves some attention. Nugis and Lamers adopt a \dot{M} -relation which is Z -dependent where, as usual, Z is defined as the abundance of all elements heavier than helium. This means that they intrinsically assume that WN and WC stars with the same luminosity and belonging to the same stellar environment (thus having a similar heavy-metal-abundance) have quite a different mass loss rate (remind that WN and WC stars have CNO abundances which differ by about a factor 100). However, there is no observational evidence that this is the case.

It is generally accepted that the WR-type stellar wind is radiation driven and therefore one may expect that *the heavy metals* are the main drivers. On that line of argument it is more plausible that the \dot{M} depends roughly on some power ζ of the heavy-metal-abundance of the

WR (which is roughly proportional to the Z -value of the hydrogen main sequence population = Z_{pop} where the WR star belongs to).

Hamann and Koesterke (2000) investigated 18 WN stars in the LMC. When these data are plotted in a \dot{M} -L diagram, there seems to be no obvious relation. However, as an exercise, Vanbeveren (2001) used a linear relation similar to (Equation 1.2) and shifted the line until the sum of the distances between observed values and predicted ones was smallest (a linear regression where the slope of the line is fixed). This resulted in a shift of -0.2 which, accounting for the Z_{pop} value of the Solar neighbourhood and of the LMC, corresponds to $\zeta = 0.5$. To be more conclusive, many more observations are needed where detailed analysis may give reliable stellar parameters, but from this exercise it looks as if *WR mass loss rates are Z_{pop} -dependent*.

The effects on massive star evolution. Due to stellar wind mass loss and/or due to Roche lobe overflow (RLOF), a massive star may become a hydrogen deficient CHeB star. The further evolution is governed by the WR-type \dot{M} . The effects on massive star evolution have been studied by Vanbeveren et al. (1998 a, b, c) (see also Vanbeveren, 2001; Van Bever and Vanbeveren, 2002). We distinguish two effects:

- Figure 1 gives the final mass of the star prior to core collapse, for single stars and for interacting binary components, for $Z=0.02$ (solar neighbourhood) and for $Z=0.002$ (appropriate for the SMC). We concluded that *with our preferred WR mass loss formalism, stars with initial mass between $40 M_{\odot}$ and $100 M_{\odot}$ end their life with a mass between $10 M_{\odot}$ and $20 M_{\odot}$* , much larger than the pre-supernova masses predicted by evolutionary calculations which were considered as a standard in the year 1997-1998 and even later. Notice that with Equation 1.2 it becomes straightforward to explain the large masses of the BH components in the high mass X-ray binary Cyg X-1 and in a number of low mass X-ray binaries.
- The \dot{M} during CHeB significantly affects the evolution of the CO-core. At the onset of CHeB, the convective CO core rapidly increases in mass. When the \dot{M} is large (as is the case for stars with an initial mass $\geq 40 M_{\odot}$ when Equation 1.2 applies), the extend of the CO-core reaches a maximum and then starts to decrease. The layers outside the decreasing CO core are carbon rich and remain so till the star's death. However, when the \dot{M} is small (as is the case when the Z_{pop} abundance is low and the \dot{M} depends on Z_{pop} as discussed in the precious paragraph), the mass of the CO core

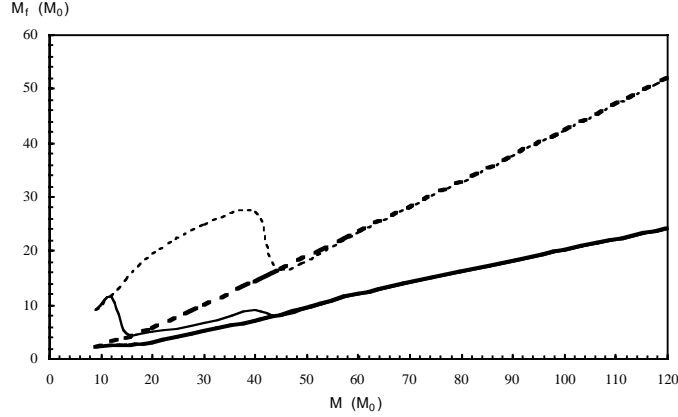


Figure 1 The final mass M_f (in M_\odot) of the star as a function of the initial mass M (in M_\odot) of the star. Full lines correspond to $Z_{pop}=0.02$ (solar neighbourhood), dashed lines correspond to $Z_{pop}=0.002$ (SMC). Thick lines hold for interacting binary components, thin lines hold for single stars.

increases till the end of CHeB. At the end of the star's life, this CO core mainly consists of O. Notice that this is also the type of CO-core evolution in massive stars which retain their hydrogen rich envelope, i.e. in single stars where the stellar wind mass loss is too small to remove the hydrogen rich layers.

In Sect. 4 we will show that the effects discussed above may be very important for chemical evolutionary simulations of the early phases of a galaxy.

4. BLACK HOLE FORMATION AND THE EFFECT ON CHEMICAL EVOLUTION

Accounting for the WR-type \dot{M} -values (Sect. 3), stellar evolution in combination with the supernova explosion simulations of Woosley and Weaver (1995) predicts that low mass BHs are descendants from single stars with an initial mass between ~ 20 - $25 M_\odot$ and ~ 35 - $40 M_\odot$, whereas all stars with initial mass $\geq 40 M_\odot$ form high mass BHs. De Donder and Vanbeveren (2003) investigated the consequences on chemical evolution simulations of the early Milky Way (early means the initial phase in the evolution of the Milky Way where the chemical enrichment of the ISM was due to massive stars only, roughly the phase where $[\text{Fe}/\text{H}] < -1$). Here we summarise some results.

One of the crucial questions in order to understand the early evolution of a galaxy is whether or not all BHs are preceded by a SN explosion. The last few years a class of extremely energetic supernova has been recognised (Nomoto et al., 2001) and it is tempting to associate them with the formation of a BH (Woosley, 1993; Paczynski, 1998; Iwamoto et al., 1998). The amount of ejected ^{56}Ni that is needed to explain these hypernova is very large, as large as $0.4\text{--}0.7 M_{\odot}$ (Sollerman et al., 2000). This means that if the formation of all BHs would be preceded by a hypernova, their contribution to the Fe enrichment of the ISM could be substantial.

A second crucial question for the early galactic evolution, related to the association of BHs and hypernovae, is whether or not the WR-like \dot{M} -values in BH-progenitors are Z_{pop} -dependent or not. As outlined in Sect. 3, when the \dot{M} is large and when the BH formation is preceded by a supernova/hypernova, the mass lost by stellar wind during CHeB and the ejected matter prior to the BH formation will contain large amounts of carbon. When the \dot{M} is small, obviously the mass lost by stellar wind during CHeB is small but the ejected matter prior to BH formation will contain large amounts of oxygen and a smaller amount of carbon.

We simulated the early galactic chemical evolution of the solar neighbourhood for the following scenario's:

- all the stars with an initial mass $\geq 40 M_{\odot}$ directly collapse into a massive BH without the ejection of matter (scen a, bold full line in Figure 2 and Figure 3)
- all the stars with an initial mass $\geq 40 M_{\odot}$ explode as a hypernova with the ejection of $0.7 M_{\odot}$ of $^{56}\text{Ni} = 0.7 M_{\odot}$ of Fe (i.e. all BHs are preceded by a hypernova) (scen b, thin full line in Figure 2 and Figure 3)
- stars with an initial mass $\geq 40 M_{\odot}$ form BHs with matter ejection, the ejected mass contains little ^{56}Ni , the WR-type \dot{M} -formalism is not Z_{pop} -dependent but satisfies Equation 1.2 so that the mass lost by stellar wind during CHeB and the ejected mass prior to the BH formation contains large amounts of carbon (scen c, dashed line in Figure 2 and Figure 3)
- stars with an initial mass $\geq 40 M_{\odot}$ form BHs with matter ejection, the ejected mass contains little ^{56}Ni , the WR-type \dot{M} -formalism is Z_{pop} -dependent and/or the single star mass loss rates are not large enough to remove the hydrogen rich layers so that the mass lost by stellar wind during CHeB and the ejected mass prior to the

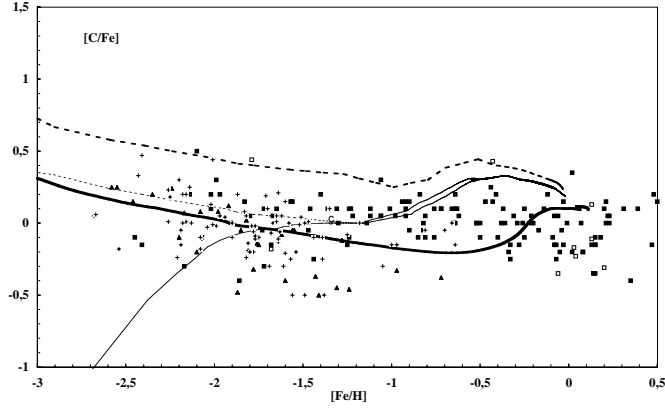


Figure 2 The simulated $[C/Fe]$ - $[Fe/H]$ relation compared to observations.

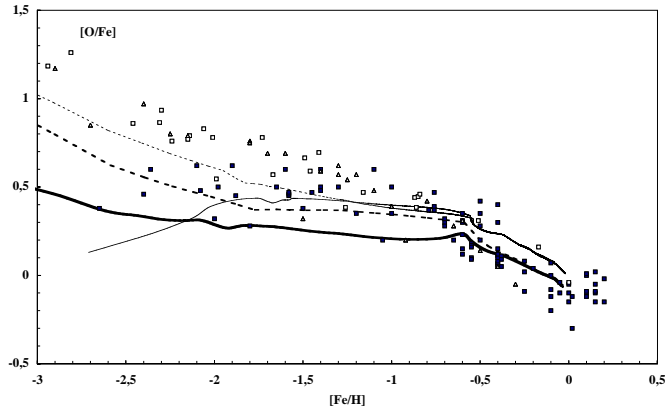


Figure 3 The simulated $[O/Fe]$ - $[Fe/H]$ relation compared to observations.

BH formation contains only a small amount of carbon but large amounts of oxygen (scen d, dotted line in Figure 2 and Figure 3).

When we compare our simulated $[O/Fe]$ - $[Fe/H]$ and $[C/Fe]$ - $[Fe/H]$ relations to observations (Figure 2 and Figure 3), we conclude:

- with scen a the predicted $[O/Fe]$ is too small
- with scen b, iron increases very rapidly. As a consequence the predicted $[O/Fe]$ and $[C/Fe]$ is always much too small, independent from the adopted WR-type M -formalism

- with scen c the predicted $[C/Fe]$ is too large; the predicted $[O/Fe]$ is larger than with scen a but it is still rather low compared to observations
- scen d is by far the best model.

A comparison between CEM simulations of the early phases of the Milky Way and observations of C, O and Fe gives indirect evidence that BH formation is rarely preceded by a (typical) hypernova explosion where large amounts of ^{56}Ni are expelled, and that mass loss by stellar wind during pre-WR phases and during the WR phase are heavy-metal-dependent

5. STELLAR ROTATION AND THE EFFECT ON CEM'S

The influence of rotation on massive single star evolution has been studied in detail by Meynet and Maeder (2000) and by Heger et al. (2000). They showed that in order to understand the evolutionary history and the chemistry of the outer layers of individual stars, the effects of rotation are very important.

To investigate the chemical evolution of galaxies, an important question is obviously in how far rotation affects average evolutionary properties.

From the studies listed above, we restrain the following evolutionary effects.

The extend of the convective core. The larger the rotation the larger the convective core during the core hydrogen burning phase and this means that convective core overshooting mimics the effect of rotation on stellar cores. The average equatorial velocity of OB-type stars ~ 100 - 150 km/sec (Penny, 1996; Vanbeveren et al., 1998a, b). When this property is linked to the calculations of Meynet and Maeder, we conclude that *the effect of rotation on the extend of the convective core during the core hydrogen phase of massive single stars is on the average similar to the effect of mild convective core overshooting as described in Schaller et al. (1992).*

Due to RLOF a binary component loses most of its hydrogen rich layers and the remnant corresponds more or less to the helium core left behind after core hydrogen burning. Rotation makes larger helium cores so that also here convective core overshooting mimics rotation.

Notice that the post-core hydrogen burning evolutionary phases are short, short enough to be able to neglect the effects of rotation on the evolution of the extend of the convective core.

Mixing of the layers outside the convective core. Rotation induces mild mixing in all mass layers outside the convective core (rotational diffusion or meridional circulation) so that some matter of the convective core can be mixed into the layers outside the convective core, even up to the outer observable envelope. Quantitatively however, the overall chemical stellar yields, expelled by the star during its entire life, which serve to enrich the ISM, hardly depend on this process. One exception: the formation of primary nitrogen in zero metallicity stars (Meynet and Maeder, 2002).

Rotation and stellar wind mass loss. The evolution of massive stars is significantly affected by stellar wind mass loss, and not in the least by the WR wind. Question: how does rotation affects the \dot{M} ? Using a theoretical hydrodynamic atmosphere model Friend and Abbott (1986) proposed the following relation:

$$\dot{M}(\omega) = \dot{M}(\omega = 0)f(\omega) \quad (1.3)$$

where $f(\omega)$ is an increasing function of the angular velocity ω . This expression is used by some scientists and included in their stellar evolutionary code (Maeder and Meynet, 2000; Heger et al., 2000). As a matter of fact, since rotation hardly affects the stellar interior structure during the post-core-hydrogen burning phases, the main effect of rotation during these post-core-hydrogen burning phases in the computations of both papers is primarily due to their adopted Equation 1.3 to describe the stellar wind mass loss rate. However, Equation 1.3 is a relation between the mass loss rate of a non-rotating star and the mass loss rate of the same star which is rotating. Maeder and Meynet and Heger et al. apply Equation 1.3 directly on the empirical mass loss rate formalisms that are used in their evolutionary code (the formalisms of Nieuwenhuijzen and de Jager, 1990, and the one of Langer, 1989). Since these empirical formalisms include data from stars that are rotating (most of them), the way Maeder and Meynet and Heger et al. study the effect of rotation on stellar wind mass loss may be wrong. Friend and Abbott themselves criticise their relation or using their own words *"...the observations (their figure 4) show that there is no evidence for a dependence of the mass loss rate on rotational velocity..."*. Even more: the relation of Friend and Abbott was seriously questioned by Owocki et al. (1996), who performed hydrodynamic simulations of the winds of rotating hot stars including the effect of nonradial radiation forces and gravity-darkening by using the von Zeipel (1924) approximation. Something similar was done in detail by Petrenz and Puls (2000) who

demonstrated that with a correct NLTE treatment of the outflowing envelope of a rotating star, the mass loss rate differs by at most 10-20 % from the mass loss rate of the same star but non-rotating, even if the star is rotating close to break-up.

Last not least: as stated already, most of the evolutionary codes rely on empirical mass loss rate formalisms. Obviously, an ideal formalism would be a formalism where the mass loss rate data are derived by an atmosphere model that includes the effects of rotation. This was done for a few stars by Petrenz and Puls (1996). An important conclusion of the latter study is that *all the mass loss rates that are determined from the H α line profiles, are overestimated typically by 20-30 % if the effects of rotation are not included*. This means that the empirical mass loss rate formalisms that were proposed in the past (and used in some stellar evolutionary codes) overestimate the effects of stellar wind mass loss.

Rotation and CEMs. Accounting for the discussion on the effects of rotation on stellar evolution in the previous paragraphs, we have made several CEM simulations with stellar chemical yields which include AVERAGE effects of rotation and we have to conclude that *the effect of rotation on general results of chemical evolutionary models of galaxies is similar to the effect of moderate convective core overshooting and is much smaller than the overall effects of binaries, with one possible exception: the formation of primary nitrogen during the very early evolution of galaxies*. Since most of the present CEMs use stellar evolutionary calculations with moderate amount of convective core overshooting, most of the CEMs already accounted for stellar rotation *avant la lettre*. Of course, there may still be evolutionary effects which have not been recognised yet.

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